Topic 3. Discounting

Future value

⇨Future Value (FV) of $100:

$$FV = \$100 \times (1+r)^t$$

Example

⇨What is the future value of $400,000 if interest is paid annually at a rate of 5% for one year?

⇨ $FV = \$400{,}000 \times (1+.05)^1 = \$420{,}000$

Present value

⇨Present Value (PV) converts future cash flows to their current values

$$PV = \text{discount factor} \times C_1$$

Discount factor

⇨Define Discount Factor (DF) as PV of $1

$$DF = \frac{1}{(1+r)^t}$$

Discount Rate

- The discount rate is the reward investors demand for accepting delayed payment.
- Investors demand what they could receive from risk-equivalent investment alternatives.
- Discount rate is also called *opportunity cost of capital* because it is the return foregone by investing in a capital project rather than investing in freely-available securities.

Net Present Value (NPV)

- Net present value is the present value of all future cash flows minus the required investment

$$\mathrm{NPV} = \frac{C_1}{1+r} - Cost$$

Example

⇨Valuing an Office Building

⇨Step 1: Forecast cash flows

– Cost of building = 370

– Sale price in Year 1: C_1 = 420

⇨Step 2: Estimate opportunity cost of capital

– If equally risky investments in the capital market offer a return of 5%, then

Cost of capital: r = 5%

⇨Step 3: Discount future cash flows

$$PV = \frac{C_1}{(1+r)} = \frac{420}{(1+.05)} = 400$$

⇨Step 4: Subtract initial cost from PV to determine if PV exceeds investment cost

$$NPV = 400 - 370 = 30$$

Net Present Value Rule

⇨Accept investments that have positive net present value

Example:

⇨Suppose we can invest $50 today and receive $60 in one year. Should we accept the project given a 10% required return?

$$NPV = -50 + \frac{60}{1.10} = \$4.55$$

Example:

⇨You can invest $100,000 today. Depending on the state of the economy, you may get one of three equally likely cash payoffs from this project:

Economy	Slump	Normal	Boom
Payoff	$80,000	110,000	140,000

⇨Should you invest given that there is a stock in the market trading for $95.65 whose next year's price is forecast at $110 with no intervening dividends?

Solution:

⇨Expected payoff of the project:

$$E(C_1) = \frac{80{,}000 + 110{,}000 + 140{,}000}{3} = \$110{,}000$$

⇨Expected return of the stock:

$$E(r_s) = \frac{\text{expected profit}}{\text{investment}} = \frac{110 - 95.65}{95.65} = .15 \text{ or } 15\%$$

⇨The stock's expected return is the (opportunity) cost of capital of the project

⇨Discounting the expected payoff at the cost of capital leads to the PV of the project:

$$PV = \frac{110{,}000}{1.15} = \$95{,}650$$

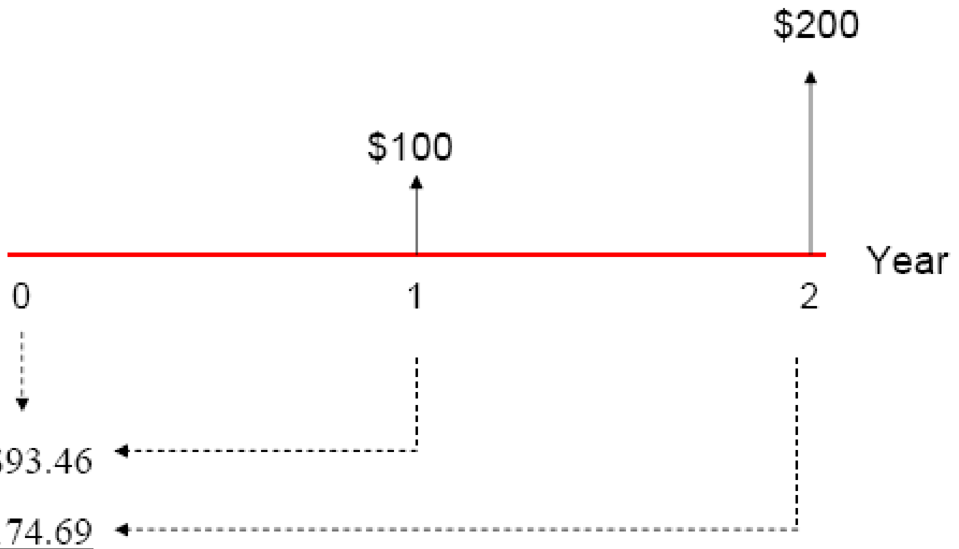
⇨Notice that you come to the same conclusion if you compare the expected project return with the cost of capital

$$E(r_p) = \frac{\text{expected profit}}{\text{investment}} = \frac{110{,}000 - 100{,}000}{100{,}000} = .10 \text{ or } 10\%$$

Multiple Cash Flows

⇨ PVs can be added together to evaluate multiple cash flows

$$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \ldots.$$



Present Value

<u>Year 0</u>

$100/1.07$	= $93.46
$200/1.07^2$	<u>= $174.69</u>
Total	= $268.15

$$PV = \frac{100}{(1+.07)^1} + \frac{200}{(1+.07)^2} = 268.15$$

Return definitions

- Realized return – return earned in the past
- Discount rate – rate used to calculate the present value of future cash flows
- Required return (fair return, hurdle rate) – rate that you require from your investments given their level of risk
- Expected return – return that you expect from your investments in the future
- Opportunity cost of capital – expected return that is *foregone* by investing in a project rather than in a comparable financial securities

  Required return, expected return and opportunity cost of capital are different names for the same concept. You discount your cash flows with the fair return or your hurdle rate which is equal to the opportunity cost of capital that you could get from alternative investments.

- Cost of capital (WACC) – cost of debt and equity financing for a company.

Perpetuity

⇨ Perpetuity: Asset that pays a level stream of cash flows in perpetuity:

$$PV = \frac{C}{r}$$

Annuity

⇨ Annuity: An asset that pays a fixed sum each year for a specified number of years

Asset	Year of Payment 1 2.....t t + 1	Present Value
Perpetuity (first payment in year 1)		$\frac{C}{r}$
Perpetuity (first payment in year t + 1)		$\left(\frac{C}{r}\right)\frac{1}{(1+r)^t}$
Annuity from year 1 to year t		$\left(\frac{C}{r}\right)-\left(\frac{C}{r}\right)\left(\frac{1}{(1+r)^t}\right)$

$$\text{PV(annuity)} = \text{PV(perpetuity)} - \frac{1}{(1+r)^t}\text{PV(perpetuity)}$$

$$\text{PV of annuity} = C \times \left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$$

Perpetuity with growth

- ⇨ Consider a security which pays C_1 next period, and then the payment grows at rate g forever
- ⇨ Let the required return be a fixed rate, r

$$PV = \frac{C_1}{r-g}$$

Annuity with growth

- ⇨ Consider a security which pays C_1 next period, and then the payments grow at rate g until period T after which it terminates
- ⇨ Equivalent to a perpetuity with growth minus the PV of a perpetuity with growth at time T

$$PV = \frac{C_1}{r-g} - \left(\frac{1+g}{1+r}\right)^T \frac{C_1}{r-g}$$

Аннуитет

Каждый месяц без всяких затей
Я получаю по «тыще» рублей.
Это не прибыль и не зарплата,
Это стипендия от деканата.
Я получаю ее пять лет.
Это и есть аннуитет.
Если б она росла с инфляцией,
Значит, была бы индексация,
Значит, это звучало бы просто:
Аннуитет с постоянным ростом.

(В. Добрынская)

Compound and simple interest

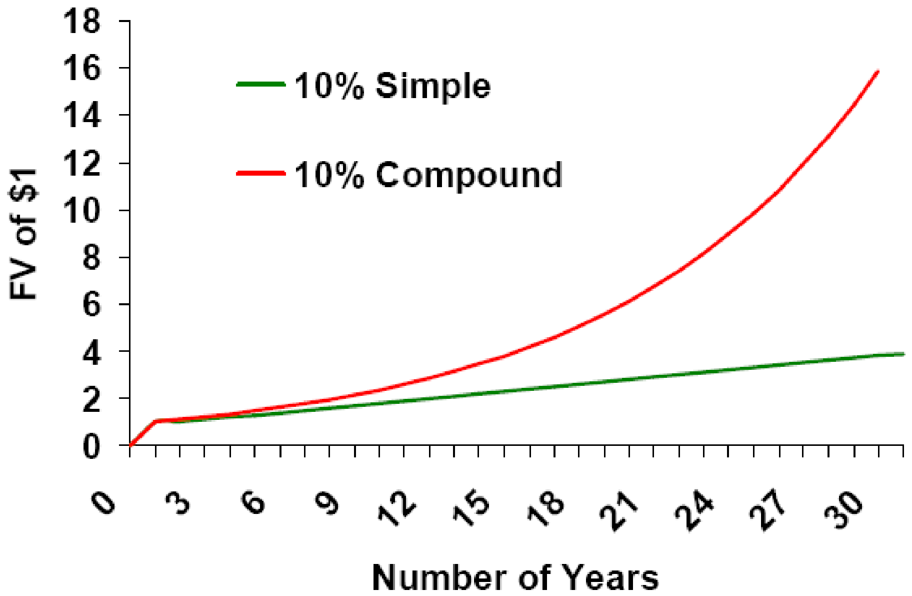
⇨ Simple interest = per period interest rate x number of periods

⇨ Compound interest = (1+per period interest rate)^(number of periods) - 1

⇨ Difference between them grows with the number of periods

Example:

i Periods per year	ii Interest per period	iii Stated Rate (i x ii)	iv Value after one year	v Annually compounded rate (APR)
1	6%	6%	1.06	6.000%
2	3	6	1.03^2 = 1.0609	6.090
4	1.5	6	1.015^4 = 1.06136	6.136
12	.5	6	1.005^{12} = 1.06168	6.168
52	.1154	6	1.001154^{52} = 1.06180	6.180
365	.0164	6	1.000164^{365} = 1.06183	6.183



Equivalent compounded rate

- An investment of $1 at a rate of r per annun compounded m times a year amounts by the end of the year to $[1+(r/m)]^m$
- The equivalent annually compounded rate of interest is $[1+(r/m)]^m-1$
- $\lim_{m\to\infty}\left[1+\frac{r}{m}\right]^m = e^r$, i.e. when the m approaches infinity, interest rate gets continuously compounded and $[1+(r/m)]^m$ approaches e^r

Nominal and Real rates

- Inflation: Rate at which general level of prices increases
- Nominal Interest Rate: Rate at which the value of investment grows in money terms
- Real Interest Rate: Rate at which the purchasing power of an investment increases

$$1 + \text{real interest rate} = \frac{1+\text{nominal interest rate}}{1+\text{inflation rate}}$$

$$(1 + \text{real interest rate}) \cdot (1 + \text{inflation rate}) = (1 + \text{nominal interest rate})$$

$$1 + \text{real rate} + \text{inflation rate} + \text{real rate} \cdot \text{inflation rate} = 1 + \text{nominal rate}$$

$$\text{real rate} + \text{inflation rate} + \text{real rate} \cdot \text{inflation rate} = \text{nominal rate}$$

$$\text{real rate} \cdot \text{inflation rate} \approx 0 \text{ (very small number if rates are low)}$$

$$\text{real interest rate} + \text{inflation rate} \approx \text{nominal interest rate}$$

$$\text{real int. rate} \approx \text{nominal int. rate} - \text{inflation rate}$$