Topic 6. Risk and return

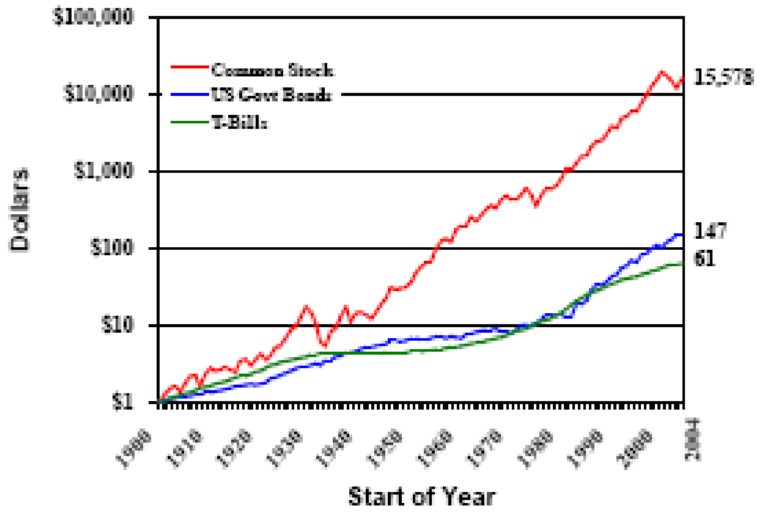
$$P = PV = \sum_{t=0}^{T} \frac{CF_t}{(1+r)^t}$$

where r – required (fair) return
given the level of risk of CF

What is the relationship between risk and return?
What is risk? How is it measured?

Capital Market History

The Value in 2004 of an Investment of $1 made in 1900, Nominal Returns



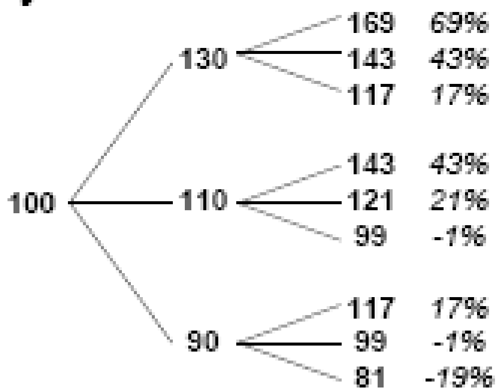
Historical Risk and Return

U.S. Historical Returns, 1926-1999

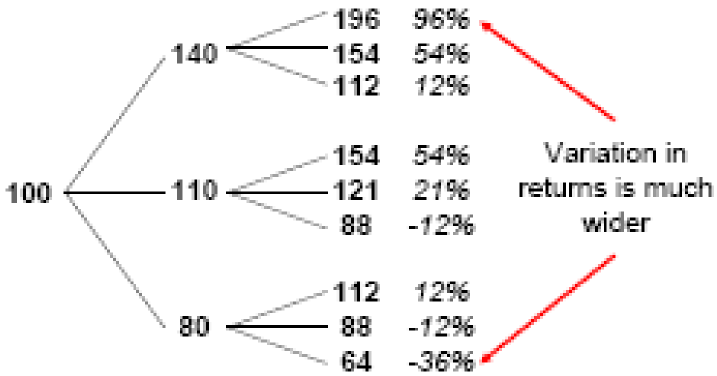
	Average Annual Return	Standard Deviation
Large Company Stocks	13.0%	20.1%
Small Company Stocks	17.7%	33.9%
Long Term Corporate Bonds	6.1%	8.7%
Long Term Government Bonds	5.6%	9.2%
US Treasury Bills	3.8%	3.2%
Inflation	3.2%	4.5%

Example

- ⇨ Suppose that the price of Big Oil's stock is $100. There is an equal chance that the stock will go up 10%, up 30% and down 10% each year over the two coming years



- ⇨ What if instead there is an equal chance that the stock will go up 40%, 10% or down 20%?
- ⇨ Expected returns: same (10%) but standard deviations different (21% vs. 42%).



Measuring Risk

- ⇨ Variance: Probability-weighted average value of squared deviations from mean. A measure of random dispersion
- ⇨ Standard Deviation: Square root of variance. A measure of dispersion with a more convenient scale

Example

- ⇨ The following table shows the nominal returns on the South African stock market and the rate of inflation

Year	Nominal return	Inflation
1977	-0.0264	.0677
1978	.0927	.0803
1979	.2556	.1331
1980	.3367	.1240
1981	-.0375	.0894

  – What was the historical standard deviation of the (nominal) market returns?

  – Calculate the average real return

Answer

- ⇨ Standard deviation is equal to:

$$\sigma = \sqrt{\frac{(-.0264-.1242)^2+(.0927-.1242)^2+(.2556-.1242)^2+(.3367-.1242)^2+(-.0375-.1242)^2}{5-1}}$$

$$=.1675=16.75\%$$

- ⇨ Average real return is equal to:

$$r=\left(\frac{1-0.0264}{1.0677}+\frac{1.0927}{1.0903}+\frac{1.2556}{1.1331}+\frac{1.3367}{1.124}+\frac{1-0.0375}{1.0894}\right)/5=0.019=1.9\%$$

Portfolio Risk

fraction of portfolio in first asset — rate of return on first asset

$$\text{Expected Portfolio Return} = (x_1 r_1) + (x_2 r_2)$$

fraction of portfolio in second asset — rate of return on second asset

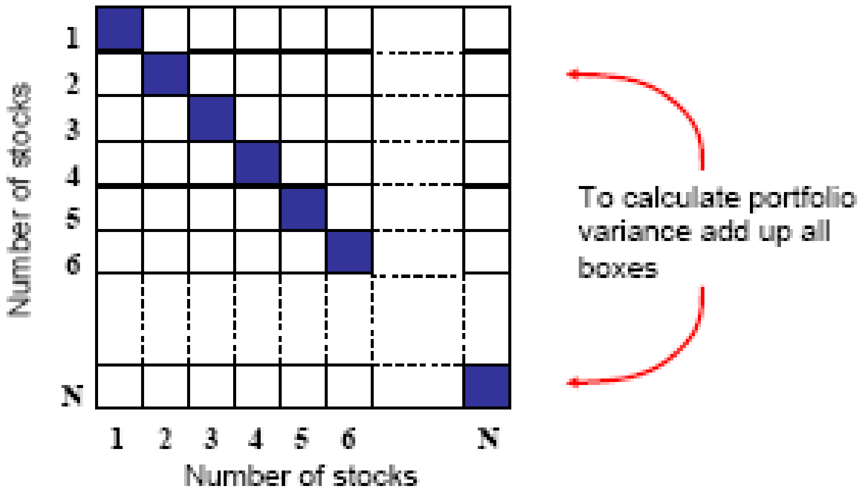
$$\text{Portfolio Variance} = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2(x_1 x_2 \rho_{12}\sigma_1\sigma_2)$$

$$\text{Portfolio Variance} = \sum_{i=1}^{m}\sum_{j=1}^{m} x_i x_j \rho_{ij}\sigma_i\sigma_j$$

⇨ The variance of a two stock portfolio is the sum of these four boxes

	Stock 1	Stock 2
Stock 1	$x_1^2\sigma_1^2$	$x_1 x_2 \sigma_{12} = x_1 x_2 \rho_{12}\sigma_1\sigma_2$
Stock 2	$x_1 x_2 \sigma_{12} = x_1 x_2 \rho_{12}\sigma_1\sigma_2$	$x_2^2\sigma_2^2$

⇨ The shaded boxes contain variance terms; the remaining contain covariance terms

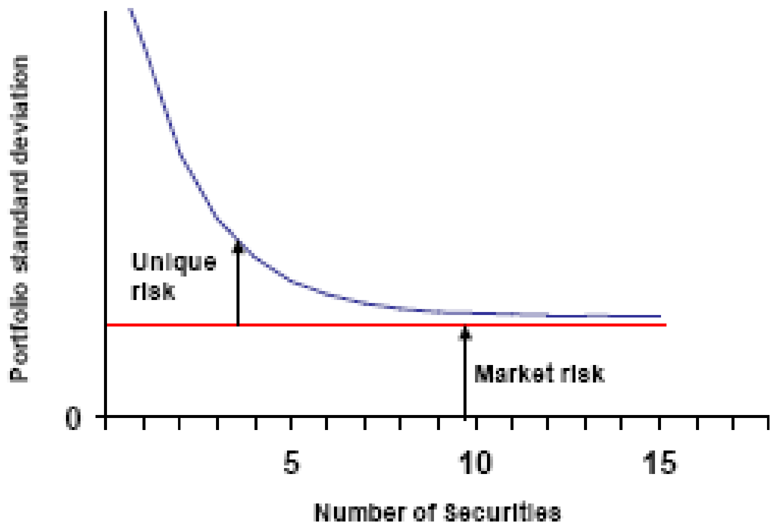


Diversification

⇨ Diversification: Strategy designed to reduce risk by spreading the portfolio across many investments

⇨ Unique Risk: Risk factors affecting only that firm. Also called “diversifiable risk”

⇨ Market Risk: Economy-wide sources of risk that affect the overall stock market. Also called “systematic risk.” Is not diversified away



An equally-weighted portfolio of N stocks:

$$\sigma_P^2 = \frac{1}{N^2}\sum_{i=1}^{N}\sigma_i^2 + \frac{1}{N^2}\sum_{i\neq j}\sigma_{ij}$$

$$\sigma_P^2 = \frac{N}{N^2}\bar{\sigma}^2 + \frac{N(N-1)}{N^2}Cov$$

$$\sigma_P^2 = \frac{1}{N}\bar{\sigma}^2 + (1-\frac{1}{N})Cov$$

As $N \to \infty$, $\sigma_P^2 \to Cov$

Диверсификация

Диверсификация – это такая операция,
Что позволяет снизить риск портфеля,
Не понеся при этом потери.

Портфель

Портфели бывают разные:
Черные, белые, красные,
Из кожи, из замши, из ткани,
Но ведь финансисты мы с вами,
И мы включаем в портфель
Облигации, акции, вексель.
И у нас портфели разные:
Консервативные и опасные,
Рыночные и дублирующие,
Диверсифицированные и хеджирующие.

(В. Добрынская)

Beta and Unique Risk

⇨ Total risk = diversifiable risk + market risk

⇨ Market risk is measured by beta, the sensitivity to market changes

$$B_i = \frac{\sigma_{im}}{\sigma_m^2}$$

Variance of the market

Covariance with the market

$$\text{Portfolio Beta} = \sum_{i=1}^{n} x_i B_i$$

⇨ Diversification lowers total risk but it does not lower beta risk, since this is "nondiversifiable" risk

True or False?

– If stocks were perfectly positively correlated, diversification would not reduce risk

– The contribution of a stock to the risk of a well-diversified portfolio depends on its market risk

Бета

Если у актива бета высока,
Значит, волатильность актива велика,
Значит, вместе с рынком движется актив,
И для сниженья риска берем дериватив,
Поскольку если бета актива высока,
Рискованность портфеля повысится слегка.

(В. Добрынская)

Asset pricing models

- **Capital Asset Pricing Model (CAPM)**

$$E(r_i) = r_f + \beta_i[E(r_M) - r_f]$$

where $$\beta_i \equiv \frac{Cov(r_i, r_M)}{Var(r_M)}$$

- **Arbitrage Pricing Theory (APT)**

$$E[r_Q] = r_f + \beta_{1Q}\lambda_1 + \beta_{2Q}\lambda_2$$

- **Fama-French three factor model**
  - ⇨ Fama and French developed a three-factor version of the APT with a good fit
  - ⇨ Factor 1: The return on the market index minus the risk-free return
  - ⇨ Factor 2: The return on a portfolio of "small" stocks minus the return on a portfolio of "large" stocks
  - ⇨ Factor 3: The return on a portfolio of "value" stocks minus the return on a portfolio of "growth" stocks

- **Four factor model with momentum**

  + Factor 4: The return on a pfl of past winners minus the return on a pfl of past loosers - *momentum* (Jegadeesh, Titman, 1993)