

Topic 9. Derivatives

Derivatives - any financial instrument whose payoff is derived from another asset's.

Derivatives with linear payoffs:

Forward – an obligation to buy/sell an underlying asset at a specified date (maturity, settlement date) and at a specified price (forward price).

- OTC = > risk of default
- Long/short positions
- Underlying assets: stocks, bonds, indices, currencies, commodities, interest rates, etc.

Futures – a standardized forward

- Exchange-traded => marked to market

Swap – an obligation to regularly exchange specified cash flows at specified dates, like a collection of forwards

- Mostly OTC
- Underlying assets: interest rates, currencies

Derivatives with non-linear payoffs:

Option – a right of a buyer to buy/sell an underlying asset at a specified date (exercise, expiration date) and at a specified price (exercise, strike price). The seller has an obligation.

- Put, call
- European, American
- Underlying assets: stocks, bonds, indices, currencies, commodities, interest rates, futures
- Prices:
 - Exercise Price (Strike Price): The price in the option contract at which you have the right to buy or sell the underlying asset
 - Option Premium: The price paid for the option
 - Intrinsic Value: Value of an option based on immediate exercise. For call option it is the maximum of zero and the difference between the strike price and the stock price.
 - Time Premium: Value of option above the intrinsic value

Embedded options (convertible bonds, callable bonds), Exotic options, Swaptions, Structured notes, Asset-backed securities (ABS), Mortgage-backed securities (MBS)

Hedging with forwards (futures)

- A farmer is currently growing 1 ton of wheat for harvest and sale in September
- The farmer needs to plan his September budget now, because of upcoming expenses
- Wheat currently sells for \$100 per ton
- Wheat prices can change unexpectedly
- A cereal maker plans to purchase 1 ton of wheat in September to make cereal
- The cereal maker needs to set its sales price for September now, for marketing purposes
- The farmer and cereal maker can BOTH hedge their September cash flows by signing a forward contract:
 - Cereal maker promises to purchase 1 ton of wheat from the farmer on September 1st at \$100
 - This is called “going long” or “buying” a forward contract
 - Farmer promises to sell the same amount and price
 - This is called “going short” or “selling” the forward contract
 - No money changes hands at contract signing
- On September 1st the spot price of wheat is \$110 per ton
- Cash settlement saves transport costs:
 - Farmer sells his wheat for \$110 to his local grainstore
 - Cereal maker purchases his wheat for \$110 from his local depot
 - The farmer sends the cereal maker a check for \$10
 - Both parties effectively “traded” at \$100, as agreed.

But there is counterparty risk (the farmer may default)!

Pricing forward (futures) contracts

Find the forward price => No-arbitrage pricing!

Two strategies:

- 1) A long position in the forward contract
- 2) Buying the underlying asset at spot and

borrowing $\frac{F_T}{(1+r_f)^T}$ at risk-free rate for T periods

Payoffs:

	t=0	t=T
Strategy 1	0	$S_T - F_T$
Strategy 2	$-S_0 + \frac{F_T}{(1+r_f)^T}$	$S_T - F_T$

Both strategies have the same risk and the same payoff at t=T. Therefore, they MUST have the

same cost => $F_T = S_0(1+r_f)^T$

If continuous compounding: $F_T = S_0 e^{r_f T}$

For interest-bearing assets: $F_T = S_0(1+r_f)^T - I_T$

or in terms of yield: $F_T = S_0(1+r_f - y)^T$, $y = I_t / S_0$

Pricing currency forwards (futures)

Find the forward exchange rate

Two strategies:

- 1) Deposit 1RUR at r_f for T periods
- 2) Convert 1RUR into USD at spot exchange rate $S_0^{RUR/USD}$, deposit the USD at r_f^* for T periods, then convert the proceeds back at forward exchange rate $F_T^{RUR/USD}$

Payoffs:

	t=0	t=T
Strategy 1	-1	$1 * (1 + r_f)^T$
Strategy 2	-1	$\frac{1}{S_0} (1 + r_f^*)^T F_T$

Both strategies are riskless and have the same cost at t=0. Therefore, they MUST have the same

payoff \Rightarrow
$$F_T = S_0 \left(\frac{1 + r_f}{1 + r_f^*} \right)^T$$

If continuous compounding:
$$F_T = S_0 e^{(r_f - r_f^*)T}$$

This is called Covered Interest Parity (CIP).